Solutions to the examination problems on Microfluidics course, 2007

1. To solve the problem we will follow a path from the point A to the point B and check the pressure changes:

$$p_A - \gamma_{water} \times 0.2m + \gamma_{oil} * h + \gamma_{water} \times 0.3m = p_B$$

The answer:

$$h = \frac{p_B - p_A + \gamma_{water} \times 0.2m - \gamma_{water} \times 0.3m}{\gamma_{oil}} = 0.449\,m$$

---

2. As $\frac{\partial \vec{V}}{\partial t} = 0$, **the local acceleration is zero**.
From the information on the velocity we can calculate the coefficients a and b:
a=15, b=10.
The velocity is:
$V = 15 \cdot x + 10\,m/s$,
and **the convective acceleration is:**
$a = 225 \cdot x + 150\,m/s^2$
at the point x=0, the acceleration a=150 m/s^2,
at the point x=1, the acceleration a=375 m/s^2.

---

3. To solve the problem we can use control volume approach. Than:
along x-axis:

$$V_1\rho\left(-V_1\right)A_1 + V_2\cos(20°)\rho V_2 A_2 = R_x + p_1 A_1 - p_2 A_2 \cos(20°)$$

Along y-axis:

$$V_1\rho\left(-V_2 \sin 20°\right)V_2\rho A_2 = R_y + p_2 A_2 \sin(20°)$$

From the known volume rate, the linear rate can be found (dividing the volume rate by the area). We find that:
V_1=3.18 m/s; V_2=12.7 m/s;
Missing pressure at the point (2) can be found from Bernoulli equation:
P_2=74.5 kPa
And now we have all the data to find the anchoring force:
R_x=-822 N
R_y=-156 N

---

4. Dividing the volumetric rate by the tube area we find the average linear velocity **V** in the tube. **V**=1.06e-3 m/s.
Now the Reynolds number can be found: **Re=1.06**.
As Re<2000 we can conclude that the flow is indeed a **laminar** one.
In the case of laminar flow you expect a parabolic profile for the velocity (MYO, p.328-329):

$$v_z = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left(r^2 - R^2\right)$$

The velocity in the middle is twice as high as the average velocity **V**.

(a) the pressure:

$$\Delta p = \frac{8\mu l V}{R^2} = 17\,\text{Pa}$$

(b) Assuming no mixing, all the liquid A will be situated within a circle of a radius R_A, which can be found as:

$$\frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left(R_A^{\,2} - R^2\right) = \frac{l}{250\,s}$$

The ratio of the areas containing A and the complete cross-section of the tube will give the percentage reading:

$$\frac{R_A^{\,2}}{R^2} = 1 - \frac{4\mu l}{250\cdot\left|\frac{\partial p}{\partial z}\right|} = 0.058$$

If liquids are inviscid they will move as a plug and A will arrive at the end of the tube after l/V=0.5/1.06e-3 = 471 s. After that moment the concentration of A will be 100%.

5. As requested in the problem we consider:

$$c = f\left(\rho, \lambda, h, \sigma\right)$$

According to the Pi-theorem we should have 5-3=2 Pi-terms. If we use h, ρ, σ as repeating variables we get:

$$\Pi_1 = c h^{1/2} \rho^{1/2} \sigma^{-1/2}$$

$$\Pi_1 = \lambda h^{-1}$$

The answer:

$$c h^{1/2} \rho^{1/2} \sigma^{-1/2} = \phi\left(\frac{\lambda}{h}\right)$$